Matchings in hypergraphs

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A matching in a graph or hypergraph is a set of disjoint edges. While finding a maximum matching in a graph is easy, it is well-known to be difficult to give good lower bounds on the maximum size of a matching in a hypergraph in terms of other natural parameters. Here we consider a hypergraph analogue of the simple fact that every regular bipartite graph of positive degree has a perfect matching. We focus on the case of tripartite hypergraphs: those for which the vertex set can be partitioned into three parts, such that each edge contains exactly one vertex from each part. We will see that every regular tripartite hypergraph with n vertices in each class has a matching of size at least n/2, and this is tight for certain special hypergraphs. We will also investigate the structure of those hypergraphs that come close to achieving this bound.